Transformer Ratio Bridges

Not only varying the impedances, but bridge can be also balanced by varying the turns ratio of a transformer. There is a small number of standard resistors and capacitors and no effect of temperature changes.
Single Ratio Transformer Bridge

Tap a transformer ⇒ voltage divider of $V_s$

$$V_1 = kN_1 = I_1Z_1 ⇒ I_1 = kN_1 / Z_1$$
$$V_2 = kN_2 = I_2Z_2 ⇒ I_2 = kN_2 / Z_2$$

To balance the bridge or no current through the detector, $D = \text{Null}$

$$I_1 = I_2 ⇒ Z_1 / Z_2 = N_1 / N_2$$

<table>
<thead>
<tr>
<th>Impedance Ratio</th>
<th>Turn Ratio</th>
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Single Ratio Transformer Bridge (Cont’d)

• Resistance Measurement
  
  $Z_1 =$ Unknown resistor $R_x$
  
  $Z_2 =$ Standard resistor $R_s$

  $$R_x = R_s \frac{N_1}{N_2}$$
Single Ratio Transformer Bridge (Cont’d)

• Capacitance Measurement

\[ Z_1 = \text{Unknown } C_x \parallel R_x \text{ (leakage resistance)} \]
\[ = \frac{1}{1/R_x + j\omega C_x} \]
\[ = \frac{R_x}{1+j\omega R_x C_x} \]

\[ Z_2 = \text{Standard } C_s \parallel R_s \]
\[ = \frac{R_s}{1+j\omega R_s C_s} \]

Single Ratio Transformer Bridge (Cont’d)

• Capacitance Measurement (Cont’d)

Balanced, \( Z_1 / Z_2 = N_1 / N_2 \)
\[ 1/Z_1 = \left(\frac{N_2}{N_1}\right) 1/Z_2 \]
\[ (1+j\omega R_x C_x)/R_x = \left(\frac{N_2}{N_1}\right) (1+j\omega R_s C_s)/R_s \]
\[ 1/R_x + j\omega C_x = \left(\frac{N_2}{N_1 R_s}\right) + j\omega C_s N_2/N_1 \]

Real part: \( R_x = R_s N_1/N_2 \)
Imagination part: \( C_x = C_s N_2/N_1 \)
Single Ratio Transformer Bridge (Cont’d)

• Inductance Measurement

\[ Z_1 = \text{Unknown } L_x \ || \ R_x \]
\[ = \frac{1}{1/R_x + 1/j\omega L_x} \]
\[ = \frac{1}{1/R_x - j/\omega L_x} \]

\[ Z_2 = \text{Standard } C_s \ || \ R_s \]
\[ = \frac{1}{1/R_s + j\omega C_s} \]
\[ = \frac{1}{1/R_s - j\omega C_s} , \text{ Reversed Current} \]

Single Ratio Transformer Bridge (Cont’d)

• Inductance Measurement (Cont’d)

Balanced, \(1/Z_1 = (N_2/N_1) 1/Z_2\)
\[ 1/R_x - j/\omega L_x = (N_2/N_1) (1/R_s - j\omega C_s) \]

Real part: \(R_x = R_s N_1/N_2\)
Imaginary part: \(1/\omega L_x = (N_2/N_1) \omega C_s\)
\[ L_x = N_1 / N_2 \omega^2 C_s \]
Double Ratio Transformer Bridge

To measure the impedance of components *in Situ*.

\[ I_1 = \frac{V_1}{Z_1} = \frac{k(N_1+N_2)}{Z_1} \]
\[ I_2 = \frac{V_2}{Z_2} = \frac{kN_2}{Z_2} \]

Double Ratio Transformer Bridge (Cont’d)

Balanced, null current or zero magnetic flux,

\[ n_1I_1 = n_2I_2 \]
\[ n_1(N_1+N_2)/Z_1 = n_2N_2/Z_2 \]
\[ Z_1 = Z_2 \frac{n_1(N_1+N_2)}{n_2N_2} \]
Q-Meter

RLC Series Resonance

\[ Z = R + j\omega L + 1/j\omega C \]

\[ = R + j(\omega L - 1/\omega C) \]

Resonant frequency

(When the voltage across C is a maximum.)

\[ \omega_0 L = 1/\omega_0 C \]

\[ \omega_0^2 = 1/LC \]

\[ \omega_0 = 1/\sqrt{LC} \]

Q-Meter (Cont’d)

\[ I_0 = V_s/R \]

\[ V_C = I_0 X_C \]

\[ = (V_s/R)(1/\omega_0 C) \]

\[ = (1/\omega_0 RC) V_s \]

\[ = Q V_s \]

\[ \propto Q \]

where \( Q = \text{Reactance/Resistance} \)

\[ = \omega_0 L / R \]

\[ = \omega_0 (1/\omega_0^2 C) / R \]

\[ = 1 / \omega_0 RC \]

(Unloaded)
Q-Meter (Cont’d)

Tuning to resonance, \( \omega_0 = 1/\sqrt{LC} \)
\[
L_x = 1 / \omega_0^2 C \\
R_x = 1 / Q\omega_0 C
\]

Q-Meter: Low Impedance Measurement

Short circuit \( \Rightarrow \) tuning \( C = C_1, L_1 = L, R_1 \)
\[
Q_1 = 1/\omega_0 R_1 C_1 \\
R_1 = 1/\omega_0 C_1 Q_1
\]
Q-Meter: Low Impedance Measurement (Cont’d)

Remove short circuit \( \Rightarrow \) then tuning \( C = C_2 \)

\[
C_2 \& C_x = C_1, \quad L_2 = L, \quad R_2 = R_1 \& R_x
\]

Series capacitors, \( C_1 = \frac{1}{1/C_x + 1/C_2} \)

\[
= \frac{C_2 C_x}{C_x + C_2}
\]

\[
C_1 C_x + C_1 C_2 = C_2 C_x
\]

\[
C_x = \frac{C_1 C_2}{C_2 - C_1}
\]

\[
Q_2 = \frac{1}{\omega_0 R_2 C_2}
\]

\[
R_2 = \frac{1}{\omega_0 C_2 Q_2}
\]

Q-Meter: Low Impedance Measurement (Cont’d)

\[
R_1 = R_2 - R_x
\]

\[
R_x = R_2 - R_1 \quad \text{, Leakage resistance}
\]

\[
= \frac{1}{\omega_0 C_2 Q_2} - \frac{1}{\omega_0 C_1 Q_1}
\]

\[
= \frac{(C_1 Q_1 - C_2 Q_2)}{(\omega_0 C_1 C_2 Q_1 Q_2)}
\]

\[
Q_x = \frac{1}{\omega_0 R_x C_x}
\]

\[
= \frac{(\omega_0 C_1 C_2 Q_1 Q_2)(C_2 - C_1)}{\omega_0 (C_1 Q_1 - C_2 Q_2) C_1 C_2}
\]

\[
= Q_1 Q_2 (C_2 - C_1) / (C_1 Q_1 - C_2 Q_2)
\]
Q-Meter: Low Impedance Measurement (Cont’d)

If the unknown component is an inductor,
\[ L_x = \frac{1}{\omega_0^2 C_x} \]
\[ = \frac{(C_2 - C_1)}{\omega_0^2 C_1 C_2} \]

If the unknown component is a pure resistor (no reactance),
\[ R_x = \frac{(C_1 Q_1 - C_2 Q_2)}{(\omega_0 C_1 C_2 Q_1 Q_2)} \]
\[ = \frac{(Q_1 - Q_2)}{\omega_0 C_1 Q_1 Q_2} \]
, \( C_1 = C_2 \)

Q-Meter: High Impedance Measurement

For high resistance, inductance > 100mH, or capacitance < 400 pF

Open circuit and tune \( C = C_1, L_1 = L, R_1 \)
Then short circuit and tune \( C = C_2 \)
\[ C_2 // C_x = C_1, L_2 = L, R_2 = R_x // R_1 \]
\[ C_x + C_2 = C_1 \quad \Rightarrow \quad C_x = C_1 - C_2 \]
\[ R_2 = R_x || R_1 = R_x R_1 / (R_x + R_1) \quad \Rightarrow \quad R_x = R_1 R_2 / (R_1 - R_2) \]
Q-Meter: High Impedance Measurement (Cont’d)

Parallel RLC ⇒ $Q = \omega_0 RC$ (loaded)
$R_1 = Q_1/\omega_0 C_1$ and $R_2 = Q_2/\omega_0 C_2$

Therefore

$R_x = Q_1 Q_2 \omega_0^2 C_1 C_2 / \omega_0^2 C_1 C_2 (Q_1 C_2 - Q_2 C_1)$

$= Q_1 Q_2 / \omega_0 (Q_1 C_2 - Q_2 C_1)$

$Q_x = \omega_0 R_x C_x$

$= \omega_0 Q_1 Q_2 (C_1 - C_2) / \omega_0 (Q_1 C_2 - Q_2 C_1)$

$= Q_1 Q_2 (C_1 - C_2) / (Q_1 C_2 - Q_2 C_1)$

For unknown inductance,

$L_x = 1/\omega_0^2 C_x$

$= 1/\omega_0^2 (C_1 - C_2)$

For pure resistance,

$R_x = Q_1 Q_2 / \omega_0 (Q_1 C_2 - Q_2 C_1)$

$= Q_1 Q_2 / \omega_0 C_1 (Q_1 - Q_2)$, $C_1 = C_2$